

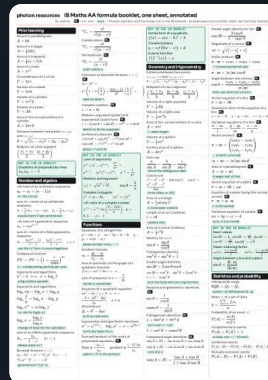
## Print it: one-page versions



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### One sheet, annotated

The same, with short pointers and the key formulas the booklet leaves out, in mint.

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Photon Resources · IB Maths AA formula booklet, annotated · [photonresources.com/ib-math-aa-formula-booklet/](https://photonresources.com/ib-math-aa-formula-booklet/)

# Prior learning

SL and HL

## Area of a parallelogram

$$A = bh$$

where  $b$  is the base,  $h$  is the height

## Area of a triangle

$$A = \frac{1}{2}(bh)$$

where  $b$  is the base,  $h$  is the height

## Area of a trapezoid

$$A = \frac{1}{2}(a + b)h$$

where  $a$  and  $b$  are the parallel sides,  $h$  is the height

## Area of a circle

$$A = \pi r^2$$

where  $r$  is the radius

## Circumference of a circle

$$C = 2\pi r$$

where  $r$  is the radius

## Volume of a cuboid

$$V = lwh$$

where  $l$  is the length,  $w$  is the width,  $h$  is the height

## Volume of a cylinder

$$V = \pi r^2 h$$

where  $r$  is the radius,  $h$  is the height

## Volume of a prism

$$V = Ah$$

where  $A$  is the area of cross-section,  $h$  is the height

## Area of the curved surface of a cylinder

$$A = 2\pi rh$$

where  $r$  is the radius,  $h$  is the height

### PHOTON NOTE

Total surface area of a closed cylinder adds the two ends:  $2\pi rh + 2\pi r^2$ .

## Distance between two points $(x_1, y_1)$ and $(x_2, y_2)$

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

## Coordinates of the midpoint of a line segment with endpoints $(x_1, y_1)$ and $(x_2, y_2)$

$$\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

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## Gradients of perpendicular lines

$$m_1 m_2 = -1$$

Use it for normals and perpendicular bisectors.

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## Topic 1: Number and algebra

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### SL 1.2 SL and HL

The  $n$ th term of an arithmetic sequence

$$u_n = u_1 + (n - 1)d$$

#### PHOTON NOTE

$(n - 1)d$ , not  $nd$ : the first term has had no differences added yet.

The sum of  $n$  terms of an arithmetic sequence

$$S_n = \frac{n}{2}(2u_1 + (n - 1)d); \quad S_n = \frac{n}{2}(u_1 + u_n)$$

### SL 1.3 SL and HL

The  $n$ th term of a geometric sequence

$$u_n = u_1 r^{n-1}$$

The sum of  $n$  terms of a finite geometric sequence

$$S_n = \frac{u_1(r^n - 1)}{r - 1} = \frac{u_1(1 - r^n)}{1 - r}, \quad r \neq 1$$

### SL 1.4 SL and HL

Compound interest

$$FV = PV \times \left(1 + \frac{r}{100k}\right)^{kn}$$

where  $FV$  is the future value,  $PV$  is the present value,  $n$  is the number of years,  $k$  is the number of compounding periods per year,  $r\%$  is the nominal annual rate of interest

#### PHOTON NOTE

On the calculator's finance solver, money you pay out is negative and money you receive is positive.

### SL 1.5 SL and HL

Exponents and logarithms

$$a^x = b \iff x = \log_a b$$

where  $a > 0$ ,  $b > 0$ ,  $a \neq 1$

### SL 1.7 SL and HL

Exponents and logarithms

$$\log_a xy = \log_a x + \log_a y$$

$$\log_a \frac{x}{y} = \log_a x - \log_a y$$

$$\log_a x^m = m \log_a x$$

#### PHOTON NOTE

There is no rule for  $\log_a(x + y)$ . Splitting it is a very common lost mark.

$$\log_a x = \frac{\log_b x}{\log_b a}$$

#### PHOTON NOTE

Change of base: lets you evaluate any log with the calculator's **ln** or **log** key.

**SL 1.8** SL and HL

The sum of an infinite geometric sequence

$$S_{\infty} = \frac{u_1}{1-r}, \quad |r| < 1$$

**PHOTON NOTE**

State the condition  $|r| < 1$  whenever you use it: it often carries a mark.

**SL 1.9** SL and HL

Binomial theorem  $n \in \mathbb{N}$

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + \dots + {}^nC_r a^{n-r}b^r + \dots + b^n$$

**PHOTON NOTE**

The general term is  ${}^nC_r a^{n-r}b^r$ . Set the power of  $x$  you want and solve for  $r$ .

$${}^nC_r = \frac{n!}{r!(n-r)!}$$

**AHL 1.10** HL only

Combinations

$${}^nC_r = \frac{n!}{r!(n-r)!}$$

**PHOTON NOTE**

Order does not matter.

Permutations

$${}^nP_r = \frac{n!}{(n-r)!}$$

**PHOTON NOTE**

Order matters.

Extension of binomial theorem,  $n \in \mathbb{Q}$

$$(a+b)^n = a^n \left( 1 + n \left( \frac{b}{a} \right) + \frac{n(n-1)}{2!} \left( \frac{b}{a} \right)^2 + \dots \right)$$

**PHOTON NOTE**

Valid only when  $\left| \frac{b}{a} \right| < 1$ . State this interval of validity.

**AHL 1.12** HL only

Complex numbers

$$z = a + bi$$

**AHL 1.13** HL only

Modulus-argument (polar) and exponential (Euler) form

$$z = r(\cos \theta + i \sin \theta) = re^{i\theta} = r \operatorname{cis} \theta$$

**PHOTON NOTE**

Find  $\theta$  from a sketch of the Argand diagram: the calculator's  $\arctan$  gives the wrong quadrant half the time.

**AHL 1.14** HL only

De Moivre's theorem

$$[r(\cos \theta + i \sin \theta)]^n = r^n(\cos n\theta + i \sin n\theta) = r^n e^{in\theta} = r^n \operatorname{cis} n\theta$$

**Laws of exponents**

$$a^m \times a^n = a^{m+n}, \quad \frac{a^m}{a^n} = a^{m-n}, \quad (a^m)^n = a^{mn}, \quad a^{-n} = \frac{1}{a^n}$$

**Modulus and argument (HL)**

$$|z| = \sqrt{a^2 + b^2}, \quad \tan \theta = \frac{b}{a}$$

Then fix the quadrant from the signs of  $a$  and  $b$ .

**Complex conjugate (HL)**

$$z^* = a - bi, \quad zz^* = |z|^2$$

Non-real roots of a real polynomial come in conjugate pairs.

 **$n$ th roots of a complex number (HL)**

$$z^{1/n} = r^{1/n} \operatorname{cis} \left( \frac{\theta + 2\pi k}{n} \right), \quad k = 0, 1, \dots, n-1$$

The  $n$  roots are equally spaced round a circle.

## Topic 2: Functions

### SL 2.1 SL and HL

Equations of a straight line

$$y = mx + c; \quad ax + by + d = 0; \quad y - y_1 = m(x - x_1)$$

Gradient formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

### SL 2.6 SL and HL

Axis of symmetry of the graph of a quadratic function

$$f(x) = ax^2 + bx + c \Rightarrow \text{axis of symmetry is } x = -\frac{b}{2a}$$

#### PHOTON NOTE

The vertex is at  $x = -\frac{b}{2a}$ ; substitute back to get its  $y$ -coordinate.

### SL 2.7 SL and HL

Solutions of a quadratic equation

$$ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \quad a \neq 0$$

Discriminant

$$\Delta = b^2 - 4ac$$

#### PHOTON NOTE

$\Delta > 0$ : two real roots;  $\Delta = 0$ : one repeated root (the graph touches the axis);  $\Delta < 0$ : no real roots.

### SL 2.9 SL and HL

Exponential and logarithmic functions

$$a^x = e^{x \ln a}; \quad \log_a a^x = x = a^{\log_a x}$$

where  $a, x > 0, a \neq 1$

### AHL 2.12 HL only

Sum and product of the roots of polynomial equations of the form  $\sum_{r=0}^n a_r x^r = 0$

$$\text{Sum is } \frac{-a_{n-1}}{a_n}; \quad \text{product is } \frac{(-1)^n a_0}{a_n}$$

#### PHOTON NOTE

$a_n$  is the leading coefficient. Watch the  $(-1)^n$  sign for odd-degree polynomials.

**Vertex form of a quadratic**

$$f(x) = a(x - h)^2 + k$$

Vertex at  $(h, k)$ . Found by completing the square.

**Transformations**

$$y = af(b(x - c)) + d$$

Vertical stretch  $a$ , horizontal stretch  $\frac{1}{b}$ , shift  $c$  right and  $d$  up. Inside the bracket acts on  $x$  the opposite way.

**Inverse function**

$$f(f^{-1}(x)) = x$$

Swap  $x$  and  $y$ , then make  $y$  the subject. The graph reflects in  $y = x$ .

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## Topic 3: Geometry and trigonometry

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### SL 3.1 SL and HL

Distance between two points  $(x_1, y_1, z_1)$  and  $(x_2, y_2, z_2)$

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

Coordinates of the midpoint of a line segment with endpoints  $(x_1, y_1, z_1)$  and  $(x_2, y_2, z_2)$

$$\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2} \right)$$

Volume of a right-pyramid

$$V = \frac{1}{3}Ah$$

where  $A$  is the area of the base,  $h$  is the height

Volume of a right cone

$$V = \frac{1}{3}\pi r^2 h$$

where  $r$  is the radius,  $h$  is the height

Area of the curved surface of a cone

$$A = \pi r l$$

where  $r$  is the radius,  $l$  is the slant height

#### PHOTON NOTE

$l$  is the slant height, not the vertical height. Find it with Pythagoras:  $l = \sqrt{r^2 + h^2}$ .

Volume of a sphere

$$V = \frac{4}{3}\pi r^3$$

where  $r$  is the radius

Surface area of a sphere

$$A = 4\pi r^2$$

where  $r$  is the radius

#### PHOTON NOTE

A hemisphere's curved surface is  $2\pi r^2$ ; add  $\pi r^2$  if it has a flat base.

### SL 3.2 SL and HL

Sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

#### PHOTON NOTE

Given two sides and a non-included angle, check for a second (obtuse) solution: the ambiguous case.

Cosine rule

$$c^2 = a^2 + b^2 - 2ab \cos C; \quad \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

#### PHOTON NOTE

Use it for three sides, or two sides and the angle between them.

Area of a triangle

$$A = \frac{1}{2}ab \sin C$$

#### PHOTON NOTE

$C$  is the angle between sides  $a$  and  $b$ .

**SL 3.4** SL and HL

Length of an arc

$$l = r\theta$$

where  $r$  is the radius,  $\theta$  is the angle measured in radians

Area of a sector

$$A = \frac{1}{2}r^2\theta$$

where  $r$  is the radius,  $\theta$  is the angle measured in radians**PHOTON NOTE**Radians only. Convert with  $180^\circ = \pi$  rad.**SL 3.5** SL and HLIdentity for  $\tan \theta$ 

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

**SL 3.6** SL and HL

Pythagorean identity

$$\cos^2 \theta + \sin^2 \theta = 1$$

Double angle identities

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta = 2 \cos^2 \theta - 1 = 1 - 2 \sin^2 \theta$$

**PHOTON NOTE**

Pick the version that leaves only one trig function in your equation.

**AHL 3.9** HL only

Reciprocal trigonometric identities

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\operatorname{cosec} \theta = \frac{1}{\sin \theta}$$

Pythagorean identities

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$$

**AHL 3.10** HL only

Compound angle identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

**PHOTON NOTE**

Note the sign flips in the cosine version.

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

Double angle identity for  $\tan$ 

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

**AHL 3.12** HL only

### Magnitude of a vector

$$|\mathbf{v}| = \sqrt{v_1^2 + v_2^2 + v_3^2}$$

$$\text{where } \mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$$

### AHL 3.13 HL only

#### Scalar product

$$\mathbf{v} \cdot \mathbf{w} = v_1w_1 + v_2w_2 + v_3w_3$$

$$\text{where } \mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}, \mathbf{w} = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$$

#### PHOTON NOTE

Perpendicular vectors have  $\mathbf{v} \cdot \mathbf{w} = 0$ .

$$\mathbf{v} \cdot \mathbf{w} = |\mathbf{v}||\mathbf{w}| \cos \theta$$

where  $\theta$  is the angle between  $\mathbf{v}$  and  $\mathbf{w}$

#### Angle between two vectors

$$\cos \theta = \frac{v_1w_1 + v_2w_2 + v_3w_3}{|\mathbf{v}||\mathbf{w}|}$$

#### PHOTON NOTE

For the angle between two lines, use their direction vectors. Take the acute angle unless told otherwise.

### AHL 3.14 HL only

#### Vector equation of a line

$$\mathbf{r} = \mathbf{a} + \lambda \mathbf{b}$$

#### PHOTON NOTE

$\mathbf{a}$  is a point on the line,  $\mathbf{b}$  its direction.

#### Parametric form of the equation of a line

$$x = x_0 + \lambda l, \quad y = y_0 + \lambda m, \quad z = z_0 + \lambda n$$

#### Cartesian equations of a line

$$\frac{x - x_0}{l} = \frac{y - y_0}{m} = \frac{z - z_0}{n}$$

### AHL 3.16 HL only

#### Vector product

$$\mathbf{v} \times \mathbf{w} = \begin{pmatrix} v_2w_3 - v_3w_2 \\ v_3w_1 - v_1w_3 \\ v_1w_2 - v_2w_1 \end{pmatrix}$$

$$\text{where } \mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}, \mathbf{w} = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$$

#### PHOTON NOTE

The result is perpendicular to both  $\mathbf{v}$  and  $\mathbf{w}$ : the quickest way to find a normal.

$$|\mathbf{v} \times \mathbf{w}| = |\mathbf{v}||\mathbf{w}| \sin \theta$$

where  $\theta$  is the angle between  $\mathbf{v}$  and  $\mathbf{w}$

### Area of a parallelogram

$$A = |\mathbf{v} \times \mathbf{w}|$$

where  $\mathbf{v}$  and  $\mathbf{w}$  form two adjacent sides of a parallelogram

#### PHOTON NOTE

A triangle with these two sides has half this area.

### AHL 3.17 **HL only**

#### Vector equation of a plane

$$\mathbf{r} = \mathbf{a} + \lambda \mathbf{b} + \mu \mathbf{c}$$

#### Equation of a plane (using the normal vector)

$$\mathbf{r} \cdot \mathbf{n} = \mathbf{a} \cdot \mathbf{n}$$

#### Cartesian equation of a plane

$$ax + by + cz = d$$

#### PHOTON NOTE

The coefficients  $(a, b, c)$  are a normal vector to the plane.

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#### Exact values

$$\sin 30^\circ = \frac{1}{2}, \cos 30^\circ = \frac{\sqrt{3}}{2}, \sin 45^\circ = \cos 45^\circ = \frac{\sqrt{2}}{2}, \tan 60^\circ = \sqrt{3}$$

Paper 1 has no calculator: learn the  $30^\circ$ ,  $45^\circ$ ,  $60^\circ$  values and their radian forms.

#### Power-reducing forms

$$\cos^2 \theta = \frac{1 + \cos 2\theta}{2}, \quad \sin^2 \theta = \frac{1 - \cos 2\theta}{2}$$

Rearranged from the booklet's  $\cos 2\theta$ . Needed to integrate  $\sin^2 x$  and  $\cos^2 x$  (HL).

#### Angle between a line and a plane (HL)

$$\sin \theta = \frac{|\mathbf{b} \cdot \mathbf{n}|}{|\mathbf{b}||\mathbf{n}|}$$

Sine, not cosine, because  $\mathbf{n}$  is perpendicular to the plane.

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## Topic 4: Statistics and probability

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### SL 4.2 SL and HL

Interquartile range

$$\text{IQR} = Q_3 - Q_1$$

#### PHOTON NOTE

An outlier is more than  $1.5 \times \text{IQR}$  below  $Q_1$  or above  $Q_3$ .

### SL 4.3 SL and HL

Mean,  $\bar{x}$ , of a set of data

$$\bar{x} = \frac{\sum_{i=1}^k f_i x_i}{n}$$

where  $n = \sum_{i=1}^k f_i$

### SL 4.5 SL and HL

Probability of an event  $A$

$$P(A) = \frac{n(A)}{n(U)}$$

Complementary events

$$P(A) + P(A') = 1$$

#### PHOTON NOTE

"At least one" questions:  $1 - P(\text{none})$ .

### SL 4.6 SL and HL

Combined events

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Mutually exclusive events

$$P(A \cup B) = P(A) + P(B)$$

#### PHOTON NOTE

Mutually exclusive means  $P(A \cap B) = 0$ . That is not the same as independent.

Conditional probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Independent events

$$P(A \cap B) = P(A)P(B)$$

#### PHOTON NOTE

To test for independence, check whether this holds with the given numbers.

### SL 4.7 SL and HL

Expected value of a discrete random variable  $X$

$$E(X) = \sum x P(X = x)$$

#### PHOTON NOTE

A fair game has  $E(\text{gain}) = 0$ .

**SL 4.8** SL and HLBinomial distribution  $X \sim B(n, p)$ **PHOTON NOTE**Four conditions: fixed  $n$ , two outcomes, constant  $p$ , independent trials. State them when asked to justify the model.

Mean

$$E(X) = np$$

Variance

$$\text{Var}(X) = np(1 - p)$$

**PHOTON NOTE**

Standard deviation is the square root of this.

**SL 4.12** SL and HL

Standardized normal variable

$$z = \frac{x - \mu}{\sigma}$$

**PHOTON NOTE**Used to find an unknown  $\mu$  or  $\sigma$ : get  $z$  from the inverse normal on the calculator.**AHL 4.13** HL only

Bayes' theorem

$$P(B|A) = \frac{P(B) P(A|B)}{P(B) P(A|B) + P(B') P(A|B')}$$

**PHOTON NOTE**

A tree diagram usually gets you there faster.

$$P(B_i|A) = \frac{P(B_i) P(A|B_i)}{P(B_1)P(A|B_1) + P(B_2)P(A|B_2) + P(B_3)P(A|B_3)}$$

**AHL 4.14** HL onlyVariance  $\sigma^2$ 

$$\sigma^2 = \frac{\sum_{i=1}^k f_i (x_i - \mu)^2}{n} = \frac{\sum_{i=1}^k f_i x_i^2}{n} - \mu^2$$

Standard deviation  $\sigma$ 

$$\sigma = \sqrt{\frac{\sum_{i=1}^k f_i (x_i - \mu)^2}{n}}$$

Linear transformation of a single random variable

$$E(aX + b) = aE(X) + b$$

$$\text{Var}(aX + b) = a^2 \text{Var}(X)$$

**PHOTON NOTE**Adding  $b$  shifts the mean but not the spread.Expected value of a continuous random variable  $X$ 

$$E(X) = \mu = \int_{-\infty}^{\infty} x f(x) dx$$

Variance

$$\text{Var}(X) = E[(X - \mu)^2] = E(X^2) - [E(X)]^2$$

Variance of a discrete random variable  $X$ 

$$\text{Var}(X) = \sum (x - \mu)^2 P(X = x) = \sum x^2 P(X = x) - \mu^2$$

Variance of a continuous random variable  $X$

$$\text{Var}(X) = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) \, dx = \int_{-\infty}^{\infty} x^2 f(x) \, dx - \mu^2$$

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**Probabilities sum to 1**

$$\sum P(X = x) = 1$$

The first step for any unknown in a discrete probability table.

**Probability density function (HL)**

$$\int_{-\infty}^{\infty} f(x) \, dx = 1, \quad P(a < X < b) = \int_a^b f(x) \, dx$$

The median  $m$  satisfies  $\int_{-\infty}^m f(x) \, dx = \frac{1}{2}$ .

## Topic 5: Calculus

### SL 5.3 SL and HL

Derivative of  $x^n$

$$f(x) = x^n \Rightarrow f'(x) = nx^{n-1}$$

#### PHOTON NOTE

Rewrite roots and fractions as powers first:  $\sqrt{x} = x^{1/2}$ ,  $\frac{1}{x^2} = x^{-2}$ .

### SL 5.5 SL and HL

Integral of  $x^n$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$$

#### PHOTON NOTE

Never drop the  $+C$  on an indefinite integral.

Area between a curve  $y = f(x)$  and the  $x$ -axis, where  $f(x) > 0$

$$A = \int_a^b y dx$$

### SL 5.6 SL and HL

Derivative of  $\sin x$

$$f(x) = \sin x \Rightarrow f'(x) = \cos x$$

#### PHOTON NOTE

Radians only.

Derivative of  $\cos x$

$$f(x) = \cos x \Rightarrow f'(x) = -\sin x$$

Derivative of  $e^x$

$$f(x) = e^x \Rightarrow f'(x) = e^x$$

Derivative of  $\ln x$

$$f(x) = \ln x \Rightarrow f'(x) = \frac{1}{x}$$

Chain rule

$$y = g(u), \text{ where } u = f(x) \Rightarrow \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

Product rule

$$y = uv \Rightarrow \frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$

Quotient rule

$$y = \frac{u}{v} \Rightarrow \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

#### PHOTON NOTE

The minus sign makes order matter:  $v u'$  comes first.

**SL 5.9** SL and HL

Acceleration

$$a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$$

Distance travelled from  $t_1$  to  $t_2$ 

$$\text{distance} = \int_{t_1}^{t_2} |v(t)| dt$$

**PHOTON NOTE**Distance uses  $|v|$ ; displacement does not. Questions ask for both.Displacement from  $t_1$  to  $t_2$ 

$$\text{displacement} = \int_{t_1}^{t_2} v(t) dt$$

**SL 5.10** SL and HL

Standard integrals

$$\int \frac{1}{x} dx = \ln |x| + C$$

$$\int \sin x dx = -\cos x + C$$

$$\int \cos x dx = \sin x + C$$

$$\int e^x dx = e^x + C$$

**SL 5.11** SL and HLArea of region enclosed by a curve and  $x$ -axis

$$A = \int_a^b |y| dx$$

**PHOTON NOTE**

If the curve crosses the axis, split the integral at the root or use the modulus on the calculator.

**AHL 5.12** HL onlyDerivative of  $f(x)$  from first principles

$$y = f(x) \Rightarrow \frac{dy}{dx} = f'(x) = \lim_{h \rightarrow 0} \left( \frac{f(x+h) - f(x)}{h} \right)$$

**PHOTON NOTE**Write "lim" on every line until you let  $h \rightarrow 0$ .**AHL 5.15** HL onlyStandard derivatives:  $\tan x$ 

$$f(x) = \tan x \Rightarrow f'(x) = \sec^2 x$$

 $\sec x$ 

$$f(x) = \sec x \Rightarrow f'(x) = \sec x \tan x$$

 $\operatorname{cosec} x$ 

$$f(x) = \operatorname{cosec} x \Rightarrow f'(x) = -\operatorname{cosec} x \cot x$$

$\cot x$

$$f(x) = \cot x \Rightarrow f'(x) = -\operatorname{cosec}^2 x$$

$a^x$

$$f(x) = a^x \Rightarrow f'(x) = a^x (\ln a)$$

$\log_a x$

$$f(x) = \log_a x \Rightarrow f'(x) = \frac{1}{x \ln a}$$

$\arcsin x$

$$f(x) = \arcsin x \Rightarrow f'(x) = \frac{1}{\sqrt{1-x^2}}$$

$\arccos x$

$$f(x) = \arccos x \Rightarrow f'(x) = -\frac{1}{\sqrt{1-x^2}}$$

$\arctan x$

$$f(x) = \arctan x \Rightarrow f'(x) = \frac{1}{1+x^2}$$

Standard integrals

$$\int a^x \, dx = \frac{1}{\ln a} a^x + C$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \arctan\left(\frac{x}{a}\right) + C$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \arcsin\left(\frac{x}{a}\right) + C, \quad |x| < a$$

### AHL 5.16 HL only

Integration by parts

$$\int u \frac{dv}{dx} \, dx = uv - \int v \frac{du}{dx} \, dx \quad \text{or} \quad \int u \, dv = uv - \int v \, du$$

#### PHOTON NOTE

Choose  $u$  as the part that gets simpler when differentiated (often a power of  $x$ , or  $\ln x$ ).

### AHL 5.17 HL only

Area of region enclosed by a curve and  $y$ -axis

$$A = \int_a^b |x| \, dy$$

Volume of revolution about the  $x$  or  $y$ -axes

$$V = \int_a^b \pi y^2 \, dx \quad \text{or} \quad V = \int_a^b \pi x^2 \, dy$$

#### PHOTON NOTE

Square first, then integrate. Limits must match the variable.

### AHL 5.18 HL only

Euler's method

$$y_{n+1} = y_n + h \times f(x_n, y_n); \quad x_{n+1} = x_n + h$$

where  $h$  is a constant (step length)

#### PHOTON NOTE

Set it up as a calculator table. Smaller  $h$  gives a better approximation.

Integrating factor for  $y' + P(x)y = Q(x)$

$$e^{\int P(x) dx}$$

PHOTON NOTE

Divide through so that  $y'$  has coefficient 1 before finding  $P(x)$ .

AHL 5.19 HL only

Maclaurin series

$$f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \dots$$

Maclaurin series for special functions

$$e^x = 1 + x + \frac{x^2}{2!} + \dots$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots$$

$$\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \dots$$

PHOTON NOTE

Substitute into these (for example  $x \rightarrow 2x$ ) rather than differentiating from scratch.

NOT IN THE IB BOOKLET · LEARN OR DERIVE THESE

Tangent and normal at  $x = a$

$$y - f(a) = f'(a)(x - a), \quad m_{\text{normal}} = -\frac{1}{f'(a)}$$

Stationary points

$$f'(x) = 0; \quad f''(x) > 0 \text{ minimum, } f''(x) < 0 \text{ maximum}$$

If  $f''(x) = 0$ , check the sign of  $f'$  either side. A point of inflexion needs  $f''$  to change sign.

Linear substitution

$$\int f(ax+b) dx = \frac{1}{a}F(ax+b) + C$$

$$\text{For example } \int e^{3x} dx = \frac{1}{3}e^{3x} + C.$$

Implicit differentiation (HL)

$$\frac{d}{dx}(y^n) = ny^{n-1} \frac{dy}{dx}$$

Differentiate every term with respect to  $x$ , then collect the  $\frac{dy}{dx}$  terms.

L'Hôpital's rule (HL)

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$$

Only for the forms  $\frac{0}{0}$  or  $\frac{\infty}{\infty}$ . Say which form it is.

Separating variables (HL)

$$\frac{dy}{dx} = f(x)g(y) \Rightarrow \int \frac{1}{g(y)} dy = \int f(x) dx$$

One  $+C$  is enough, on one side.

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