

Prior learning

Area of a parallelogram
 $A = bh$
Area of a triangle
 $A = \frac{1}{2}(bh)$
Area of a trapezoid
 $A = \frac{1}{2}(a + b)h$
Area of a circle
 $A = \pi r^2$
Circumference of a circle
 $C = 2\pi r$
Volume of a cuboid
 $V = lwh$
Volume of a cylinder
 $V = \pi r^2 h$
Volume of a prism
 $V = Ah$
Area of the curved surface of a cylinder
 $A = 2\pi rh$
Distance between two points (x_1, y_1) and (x_2, y_2)
 $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
Midpoint of a line segment
 $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

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Gradients of perpendicular lines
 $m_1 m_2 = -1$

Number and algebra

n th term of an arithmetic sequence
 $u_n = u_1 + (n - 1)d$
($n-1$)d, not nd
sum of n terms of an arithmetic sequence
 $S_n = \frac{n}{2}(2u_1 + (n - 1)d)$; $S_n = \frac{n}{2}(u_1 + u_n)$
second form if last term known
 n th term of a geometric sequence
 $u_n = u_1 r^{n-1}$
sum of n terms of a finite geometric sequence
 $S_n = \frac{u_1(r^n - 1)}{r - 1} = \frac{u_1(1 - r^n)}{1 - r}$, $r \neq 1$
use the $r > 1$ form to avoid negatives
Compound interest
 $FV = PV \times \left(1 + \frac{r}{100k}\right)^{kn}$
 k = compounding periods per year
Exponents and logarithms
 $a^x = b \iff x = \log_a b$
a log undoes a power
Exponents and logarithms
 $\log_a xy = \log_a x + \log_a y$
 $\log_a \frac{x}{y} = \log_a x - \log_a y$
 $\log_a x^m = m \log_a x$
no rule for $\log(x+y)$
 $\log_a x = \frac{\log_b x}{\log_b a}$
change of base for the calculator
sum of an infinite geometric sequence
 $S_\infty = \frac{u_1}{1 - r}$, $|r| < 1$
always state $|r| < 1$

Binomial theorem $n \in \mathbb{N}$
 $(a + b)^n = a^n + {}^nC_1 a^{n-1} b + \dots + {}^nC_r a^{n-r} b^r + \dots + b^n$
general term ${}^nC_r a^{n-r} b^r$
 ${}^nC_r = \frac{n!}{r!(n - r)!}$
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Laws of exponents
 $a^m \times a^n = a^{m+n}$, $\frac{a^m}{a^n} = a^{m-n}$, $(a^m)^n = a^{mn}$, $a^{-n} = \frac{1}{a^n}$

Functions

Equations of a straight line
 $y = mx + c$; $ax + by + d = 0$; $y - y_1 = m(x - x_1)$
perpendicular: $m_1 m_2 = -1$
Gradient formula
 $m = \frac{y_2 - y_1}{x_2 - x_1}$
Axis of symmetry of the graph of a quadratic function
 $f(x) = ax^2 + bx + c \Rightarrow$
axis of symmetry is $x = -\frac{b}{2a}$
vertex x-coordinate
Solutions of a quadratic equation
 $ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, $a \neq 0$
Discriminant
 $\Delta = b^2 - 4ac$
 $\Delta < 0$: no real roots
Exponential and logarithmic functions
 $a^x = e^{x \ln a}$; $\log_a a^x = x = a^{\log_a x}$
turns any base into e

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Vertex form of a quadratic
 $f(x) = a(x - h)^2 + k$
Transformations
 $y = af(b(x - c)) + d$
Inverse function
 $f(f^{-1}(x)) = x$

Geometry and trigonometry

Distance between two points
 (x_1, y_1, z_1) and (x_2, y_2, z_2)
 $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$
Midpoint of a line segment
 $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$
Volume of a right-pyramid
 $V = \frac{1}{3}Ah$
Volume of a right cone
 $V = \frac{1}{3}\pi r^2 h$
Area of the curved surface of a cone
 $A = \pi r l$
 l = slant height
Volume of a sphere
 $V = \frac{4}{3}\pi r^3$
Surface area of a sphere
 $A = 4\pi r^2$
Sine rule
 $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
check the ambiguous case
Cosine rule
 $c^2 = a^2 + b^2 - 2ab \cos C$; $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$
three sides, or SAS

Area of a triangle
 $A = \frac{1}{2}ab \sin C$
 C is between a and b
Length of an arc (radians)
 $l = r\theta$
radians only
Area of a sector (radians)
 $A = \frac{1}{2}r^2 \theta$
Identity for $\tan \theta$
 $\tan \theta = \frac{\sin \theta}{\cos \theta}$
Pythagorean identity
 $\cos^2 \theta + \sin^2 \theta = 1$
Double angle identities
 $\sin 2\theta = 2 \sin \theta \cos \theta$
 $\cos 2\theta = \cos^2 \theta - \sin^2 \theta = 2 \cos^2 \theta - 1 = 1 - 2 \sin^2 \theta$
pick the form with one trig function

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Exact values
 $\sin 30^\circ = \frac{1}{2}$, $\cos 30^\circ = \frac{\sqrt{3}}{2}$, $\sin 45^\circ = \cos 45^\circ = \frac{\sqrt{2}}{2}$, $\tan 60^\circ = \sqrt{3}$
Power-reducing forms
 $\cos^2 \theta = \frac{1 + \cos 2\theta}{2}$, $\sin^2 \theta = \frac{1 - \cos 2\theta}{2}$

Statistics and probability

Interquartile range
 $IQR = Q_3 - Q_1$
outlier: $1.5 \times IQR$ beyond Q_1, Q_3
Mean, $\bar{x}$, of a set of data
 $\bar{x} = \frac{\sum_{i=1}^k f_i x_i}{n}$
Probability of an event A
 $P(A) = \frac{n(A)}{n(U)}$
Complementary events
 $P(A) + P(A') = 1$
at least one = $1 - P(\text{none})$
Combined events
 $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
Mutually exclusive events
 $P(A \cup B) = P(A) + P(B)$
Conditional probability
 $P(A|B) = \frac{P(A \cap B)}{P(B)}$
given B : divide by $P(B)$
Independent events
 $P(A \cap B) = P(A)P(B)$
use it to test independence
Expected value of a discrete random variable X
 $E(X) = \sum x P(X = x)$
fair game: $E = 0$
Binomial distribution $X \sim B(n, p)$
Mean
 $E(X) = np$
Variance
 $\text{Var}(X) = np(1 - p)$
 $SD = \sqrt{\text{variance}}$
Standardized normal variable
 $z = \frac{x - \mu}{\sigma}$
inverse normal for unknown μ, σ

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Probabilities sum to 1
 $\sum P(X = x) = 1$

Calculus

Derivative of x^n
 $f(x) = x^n \Rightarrow f'(x) = nx^{n-1}$
roots and fractions $\rightarrow$ powers
Integral of x^n
 $\int x^n dx = \frac{x^{n+1}}{n+1} + C$, $n \neq -1$
never drop $+C$
Area between a curve $y = f(x)$ and the x -axis,
 $A = \int_a^b y dx$
Derivative of $\sin x$
 $f(x) = \sin x \Rightarrow f'(x) = \cos x$
Derivative of $\cos x$
 $f(x) = \cos x \Rightarrow f'(x) = -\sin x$
Derivative of e^x
 $f(x) = e^x \Rightarrow f'(x) = e^x$
Derivative of $\ln x$
 $f(x) = \ln x \Rightarrow f'(x) = \frac{1}{x}$
Chain rule
 $y = g(u)$, where $u = f(x) \Rightarrow \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
outer' $\times$ inner'
Product rule
 $y = uv \Rightarrow \frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$
Quotient rule
 $y = \frac{u}{v} \Rightarrow \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$
v-u' comes first
Acceleration
 $a = \frac{dv}{dt} = \frac{d^2 s}{dt^2}$
Distance travelled from t_1 to t_2
distance = $\int_{t_1}^{t_2} |v(t)| dt$
distance uses $|v|$
Displacement from t_1 to t_2
displacement = $\int_{t_1}^{t_2} v(t) dt$
Standard integrals
 $\int \frac{1}{x} dx = \ln |x| + C$
 $\int \sin x dx = -\cos x + C$
 $\int \cos x dx = \sin x + C$
 $\int e^x dx = e^x + C$
Area of region enclosed by a curve and x -axis
 $A = \int_a^b |y| dx$
split at the roots
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Tangent and normal at $x = a$
 $y - f(a) = f'(a)(x - a)$
 $a)$, $m_{\text{normal}} = -\frac{1}{f'(a)}$
Stationary points
 $f'(x) = 0$; $f''(x) > 0$ minimum, $f''(x) < 0$ maximum
Linear substitution
 $\int f(ax + b) dx = \frac{1}{a} F(ax + b) + C$

