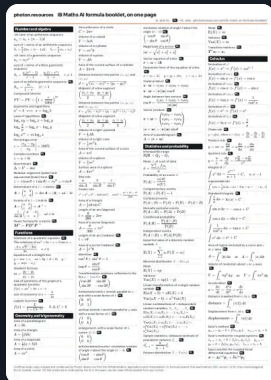


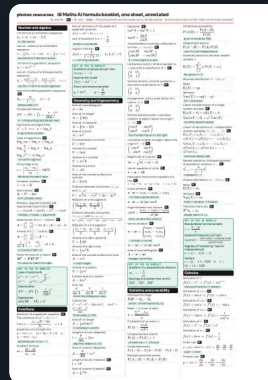
Print it: one-page versions



On one page

Every official formula on a single A4 side. Print it, pin it up.

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One sheet, annotated

The same, with short pointers and the key formulas the booklet leaves out, in mint.

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SL student? Skip the HL lines: SL-only [one page](#) · [annotated](#) (free PDFs). [All booklets](#)

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Topic 1: Number and algebra

1.2 SL and HL

The n th term of an arithmetic sequence

$$u_n = u_1 + (n - 1)d$$

PHOTON NOTE

$(n - 1)d$, not nd : the first term has had no differences added yet.

The sum of n terms of an arithmetic sequence

$$S_n = \frac{n}{2}(2u_1 + (n - 1)d); \quad S_n = \frac{n}{2}(u_1 + u_n)$$

1.3 SL and HL

The n th term of a geometric sequence

$$u_n = u_1 r^{n-1}$$

The sum of n terms of a finite geometric sequence

$$S_n = \frac{u_1(r^n - 1)}{r - 1} = \frac{u_1(1 - r^n)}{1 - r}, \quad r \neq 1$$

1.11 HL only

The sum of an infinite geometric sequence

$$S_\infty = \frac{u_1}{1 - r}, \quad |r| < 1$$

PHOTON NOTE

State the condition $|r| < 1$ whenever you use it: it often carries a mark.

1.4 SL and HL

Compound interest

$$FV = PV \times \left(1 + \frac{r}{100k}\right)^{kn}$$

where FV is the future value, PV is the present value, n is the number of years, k is the number of compounding periods per year, $r\%$ is the nominal annual rate of interest

PHOTON NOTE

On the calculator's finance solver, money you pay out is negative and money you receive is positive.

1.5 SL and HL

Exponents and logarithms

$$a^x = b \iff x = \log_a b$$

where $a > 0$, $b > 0$, $a \neq 1$

1.9 HL only

Laws of logarithms

$$\log_a xy = \log_a x + \log_a y$$

$$\log_a \frac{x}{y} = \log_a x - \log_a y$$

$$\log_a x^m = m \log_a x$$

for $a, x, y > 0$

PHOTON NOTE

There is no rule for $\log_a(x + y)$. Splitting it is a very common lost mark.

1.6 SL and HL

Percentage error

$$\varepsilon = \left| \frac{v_A - v_E}{v_E} \right| \times 100\%$$

where v_E is the exact value and v_A is the approximate value of v

PHOTON NOTE

Divide by the exact value, not the approximate one.

1.12 HL only

Complex numbers

$$z = a + bi$$

Discriminant

$$\Delta = b^2 - 4ac$$

PHOTON NOTE

$\Delta < 0$ gives two complex conjugate roots.

1.13 HL only

Modulus-argument (polar) and exponential (Euler) form

$$z = r(\cos \theta + i \sin \theta) = re^{i\theta} = r \operatorname{cis} \theta$$

PHOTON NOTE

Multiply: multiply the moduli, add the arguments.

1.14 HL only

Determinant of a 2×2 matrix

$$\mathbf{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \Rightarrow \det \mathbf{A} = |\mathbf{A}| = ad - bc$$

Inverse of a 2×2 matrix

$$\mathbf{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \Rightarrow \mathbf{A}^{-1} = \frac{1}{\det \mathbf{A}} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}, \quad ad \neq bc$$

PHOTON NOTE

Swap the leading diagonal, negate the other one. No inverse when the determinant is 0.

1.15 HL only

Power formula for a matrix

$$\mathbf{M}^n = \mathbf{P} \mathbf{D}^n \mathbf{P}^{-1}$$

where $\mathbf{P}$ is the matrix of eigenvectors and $\mathbf{D}$ is the diagonal matrix of eigenvalues

PHOTON NOTE

Columns of $\mathbf{P}$ and entries of $\mathbf{D}$ must be in the same order.

Laws of exponents

$$a^m \times a^n = a^{m+n}, \quad \frac{a^m}{a^n} = a^{m-n}, \quad (a^m)^n = a^{mn}$$

Depreciation

$$FV = PV \left(1 - \frac{r}{100}\right)^n$$

Depreciation at $r\%$ a year. Loans and annuities go through the calculator's finance solver.

Eigenvalues (HL)

$$\det(\mathbf{M} - \lambda \mathbf{I}) = 0$$

Solve this characteristic equation for λ , then find each eigenvector from $(\mathbf{M} - \lambda \mathbf{I})\mathbf{v} = \mathbf{0}$.

Topic 2: Functions

Prior learning **HL only**

Solutions of a quadratic equation

The solutions of $ax^2 + bx + c = 0$ are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, $a \neq 0$

2.1 **SL and HL**

Equations of a straight line

$$y = mx + c; \quad ax + by + d = 0; \quad y - y_1 = m(x - x_1)$$

Gradient formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

2.5 **SL and HL**

Axis of symmetry of the graph of a quadratic function

$$f(x) = ax^2 + bx + c \Rightarrow \text{axis of symmetry is } x = -\frac{b}{2a}$$

PHOTON NOTE

The vertex is at $x = -\frac{b}{2a}$; substitute back to get its y -coordinate.

2.9 **HL only**

Logistic function

$$f(x) = \frac{L}{1 + Ce^{-kx}}, \quad L, k, C > 0$$

PHOTON NOTE

L is the carrying capacity: the horizontal asymptote the model levels off at.

NOT IN THE IB BOOKLET · LEARN OR DERIVE THESE

Gradients of perpendicular lines

$$m_1 m_2 = -1$$

Needed for perpendicular bisectors in Voronoi diagrams.

Exponential model

$$f(x) = ka^x + c$$

c is the horizontal asymptote. Growth if $a > 1$, decay if $0 < a < 1$.

Direct and inverse variation

$$y = kx^n, \quad y = \frac{k}{x^n}$$

Topic 3: Geometry and trigonometry

Prior learning SL and HL

Area of a parallelogram

$$A = bh$$

where b is the base, h is the height

Area of a triangle

$$A = \frac{1}{2}(bh)$$

where b is the base, h is the height

Area of a trapezoid

$$A = \frac{1}{2}(a + b)h$$

where a and b are the parallel sides, h is the height

Area of a circle

$$A = \pi r^2$$

where r is the radius

Circumference of a circle

$$C = 2\pi r$$

where r is the radius

Volume of a cuboid

$$V = lwh$$

where l is the length, w is the width, h is the height

Volume of a cylinder

$$V = \pi r^2 h$$

where r is the radius, h is the height

Volume of a prism

$$V = Ah$$

where A is the area of cross-section, h is the height

Area of the curved surface of a cylinder

$$A = 2\pi rh$$

where r is the radius, h is the height

PHOTON NOTE

Total surface area of a closed cylinder adds the two ends: $2\pi rh + 2\pi r^2$.

Distance between two points (x_1, y_1) and (x_2, y_2)

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Coordinates of the midpoint of a line segment with endpoints (x_1, y_1) and (x_2, y_2)

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

3.1 SL and HL

Distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2)

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

Coordinates of the midpoint of a line segment with endpoints (x_1, y_1, z_1) and (x_2, y_2, z_2)

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2} \right)$$

Volume of a right-pyramid

$$V = \frac{1}{3}Ah$$

where A is the area of the base, h is the height

Volume of a right cone

$$V = \frac{1}{3}\pi r^2 h$$

where r is the radius, h is the height

Area of the curved surface of a cone

$$A = \pi r l$$

where r is the radius, l is the slant height

PHOTON NOTE

l is the slant height, not the vertical height. Find it with Pythagoras: $l = \sqrt{r^2 + h^2}$.

Volume of a sphere

$$V = \frac{4}{3}\pi r^3$$

where r is the radius

Surface area of a sphere

$$A = 4\pi r^2$$

where r is the radius

PHOTON NOTE

A hemisphere's curved surface is $2\pi r^2$; add πr^2 if it has a flat base.

3.2 SL and HL

Sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

PHOTON NOTE

Given two sides and a non-included angle, check for a second (obtuse) solution: the ambiguous case.

Cosine rule

$$c^2 = a^2 + b^2 - 2ab \cos C; \quad \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

PHOTON NOTE

Use it for three sides, or two sides and the angle between them.

Area of a triangle

$$A = \frac{1}{2}ab \sin C$$

PHOTON NOTE

C is the angle between sides a and b .

3.4 SL and HL

Length of an arc

$$l = \frac{\theta}{360} \times 2\pi r$$

where θ is the angle measured in degrees, r is the radius

Area of a sector

$$A = \frac{\theta}{360} \times \pi r^2$$

where θ is the angle measured in degrees, r is the radius

PHOTON NOTE

SL uses degrees here; HL adds the radian versions in 3.7.

3.7 HL only

Length of an arc

$$l = r\theta$$

where r is the radius, θ is the angle measured in radians

Area of a sector

$$A = \frac{1}{2}r^2\theta$$

where r is the radius, θ is the angle measured in radians

3.8 HL only

Identities

$$\cos^2 \theta + \sin^2 \theta = 1$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

3.9 HL only

Transformation matrices

$$\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$$

reflection in the line $y = (\tan \theta)x$

PHOTON NOTE

Use θ , the angle the line makes with the x -axis; the matrix doubles it for you.

$$\begin{pmatrix} k & 0 \\ 0 & 1 \end{pmatrix}$$

horizontal stretch / stretch parallel to x -axis with a scale factor of k

$$\begin{pmatrix} 1 & 0 \\ 0 & k \end{pmatrix}$$

vertical stretch / stretch parallel to y -axis with a scale factor of k

$$\begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}$$

enlargement, with a scale factor of k , centre $(0, 0)$

$$\begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$$

anticlockwise/counter-clockwise rotation of angle θ about the origin ($\theta > 0$)

$$\begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$$

clockwise rotation of angle θ about the origin ($\theta > 0$)

PHOTON NOTE

For "A then B", multiply **BA**: the first transformation goes on the right. Area scales by $|\det|$.

3.10 HL only

Magnitude of a vector

$$|\mathbf{v}| = \sqrt{v_1^2 + v_2^2 + v_3^2}$$

where $\mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$

3.11 HL only

Vector equation of a line

$$\mathbf{r} = \mathbf{a} + \lambda \mathbf{b}$$

PHOTON NOTE

In kinematics questions, $\mathbf{b}$ is the velocity and λ is time.

Parametric form of the equation of a line

$$x = x_0 + \lambda l, \quad y = y_0 + \lambda m, \quad z = z_0 + \lambda n$$

3.13 HL only

Scalar product

$$\mathbf{v} \cdot \mathbf{w} = v_1 w_1 + v_2 w_2 + v_3 w_3$$

where $\mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}, \mathbf{w} = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$

PHOTON NOTE

Perpendicular vectors have $\mathbf{v} \cdot \mathbf{w} = 0$.

$$\mathbf{v} \cdot \mathbf{w} = |\mathbf{v}| |\mathbf{w}| \cos \theta$$

where θ is the angle between $\mathbf{v}$ and $\mathbf{w}$

Angle between two vectors

$$\cos \theta = \frac{v_1 w_1 + v_2 w_2 + v_3 w_3}{|\mathbf{v}| |\mathbf{w}|}$$

PHOTON NOTE

For the angle between two lines, use their direction vectors. Take the acute angle unless told otherwise.

Vector product

$$\mathbf{v} \times \mathbf{w} = \begin{pmatrix} v_2 w_3 - v_3 w_2 \\ v_3 w_1 - v_1 w_3 \\ v_1 w_2 - v_2 w_1 \end{pmatrix}$$

where $\mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}, \mathbf{w} = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$

PHOTON NOTE

The result is perpendicular to both $\mathbf{v}$ and $\mathbf{w}$: the quickest way to find a normal.

$$|\mathbf{v} \times \mathbf{w}| = |\mathbf{v}| |\mathbf{w}| \sin \theta$$

where θ is the angle between $\mathbf{v}$ and $\mathbf{w}$

Area of a parallelogram

$$A = |\mathbf{v} \times \mathbf{w}|$$

where $\mathbf{v}$ and $\mathbf{w}$ form two adjacent sides of a parallelogram

PHOTON NOTE

A triangle with these two sides has half this area.

Gradient of a perpendicular bisector

$$m_{\perp} = -\frac{1}{m}$$

Through the midpoint of the two sites. This builds every Voronoi edge.

Bearings (clockwise from north)

045°, 120°, 300°

Always three figures: 045°, not 45°.

Topic 4: Statistics and probability

4.2 SL and HL

Interquartile range

$$\text{IQR} = Q_3 - Q_1$$

PHOTON NOTE

An outlier is more than $1.5 \times \text{IQR}$ below Q_1 or above Q_3 .

4.3 SL and HL

Mean, $\bar{x}$, of a set of data

$$\bar{x} = \frac{\sum_{i=1}^k f_i x_i}{n}$$

where $n = \sum_{i=1}^k f_i$

4.5 SL and HL

Probability of an event A

$$P(A) = \frac{n(A)}{n(U)}$$

Complementary events

$$P(A) + P(A') = 1$$

PHOTON NOTE

"At least one" questions: $1 - P(\text{none})$.

4.6 SL and HL

Combined events

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Mutually exclusive events

$$P(A \cup B) = P(A) + P(B)$$

PHOTON NOTE

Mutually exclusive means $P(A \cap B) = 0$. That is not the same as independent.

Conditional probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Independent events

$$P(A \cap B) = P(A)P(B)$$

PHOTON NOTE

To test for independence, check whether this holds with the given numbers.

4.7 SL and HL

Expected value of a discrete random variable X

$$E(X) = \sum_{i=1}^k x_i P(X = x_i)$$

PHOTON NOTE

A fair game has $E(\text{gain}) = 0$.

4.8 SL and HL

Binomial distribution $X \sim B(n, p)$

PHOTON NOTE

Four conditions: fixed n , two outcomes, constant p , independent trials. State them when asked to justify the model.

Mean

$$E(X) = np$$

Variance

$$\text{Var}(X) = np(1 - p)$$

PHOTON NOTE

Standard deviation is the square root of this.

4.14 HL only

Linear transformation of a single random variable

$$E(aX + b) = aE(X) + b$$

$$\text{Var}(aX + b) = a^2 \text{Var}(X)$$

Linear combinations of n independent random variables, $X_1, X_2, \dots, X_n$

$$E(a_1X_1 \pm a_2X_2 \pm \dots \pm a_nX_n) = a_1E(X_1) \pm a_2E(X_2) \pm \dots \pm a_nE(X_n)$$

$$\text{Var}(a_1X_1 \pm a_2X_2 \pm \dots \pm a_nX_n) = a_1^2\text{Var}(X_1) + a_2^2\text{Var}(X_2) + \dots + a_n^2\text{Var}(X_n)$$

PHOTON NOTE

Variances always add, even when the variables are subtracted. And $\text{Var}(2X) \neq \text{Var}(X_1 + X_2)$.

Sample statistics. Unbiased estimate of population variance s_{n-1}^2

$$s_{n-1}^2 = \frac{n}{n-1} s_n^2$$

4.17 HL only

Poisson distribution $X \sim \text{Po}(m)$

PHOTON NOTE

Events occur singly, independently and at a constant average rate. Scale m to the time interval asked about.

Mean

$$E(X) = m$$

Variance

$$\text{Var}(X) = m$$

PHOTON NOTE

Mean $\approx$ variance in the data suggests a Poisson model.

4.19 HL only

Transition matrices

$$\mathbf{T}^n \mathbf{s}_0 = \mathbf{s}_n$$

where $\mathbf{s}_0$ is the initial state

PHOTON NOTE

The steady state $\mathbf{s}$ solves $\mathbf{T}\mathbf{s} = \mathbf{s}$, with entries adding to 1.

Standardized normal variable

$$z = \frac{x - \mu}{\sigma}$$

Not in the AI booklet. Needed to find an unknown μ or σ .

Expected frequency (χ^2 test)

$$\text{expected} = \frac{\text{row total} \times \text{column total}}{\text{grand total}}$$

Every expected frequency should be at least 5; combine classes if not.

Degrees of freedom (χ^2 test for independence)

$$\nu = (r - 1)(c - 1)$$

For a goodness of fit test: number of classes - 1 (minus any parameters estimated).

Outliers

$$x < Q_1 - 1.5 \times \text{IQR} \quad \text{or} \quad x > Q_3 + 1.5 \times \text{IQR}$$

Topic 5: Calculus

5.3 SL and HL

Derivative of x^n

$$f(x) = x^n \Rightarrow f'(x) = nx^{n-1}$$

PHOTON NOTE

Rewrite roots and fractions as powers first: $\sqrt{x} = x^{1/2}$, $\frac{1}{x^2} = x^{-2}$.

5.9 HL only

Derivative of $\sin x$

$$f(x) = \sin x \Rightarrow f'(x) = \cos x$$

Derivative of $\cos x$

$$f(x) = \cos x \Rightarrow f'(x) = -\sin x$$

Derivative of $\tan x$

$$f(x) = \tan x \Rightarrow f'(x) = \frac{1}{\cos^2 x}$$

Derivative of e^x

$$f(x) = e^x \Rightarrow f'(x) = e^x$$

Derivative of $\ln x$

$$f(x) = \ln x \Rightarrow f'(x) = \frac{1}{x}$$

Chain rule

$$y = g(u), \text{ where } u = f(x) \Rightarrow \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

Product rule

$$y = uv \Rightarrow \frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$

Quotient rule

$$y = \frac{u}{v} \Rightarrow \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

5.5 SL and HL

Integral of x^n

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$$

PHOTON NOTE

Never drop the $+C$ on an indefinite integral.

5.8 SL and HL

The trapezoidal rule

$$\int_a^b y \, dx \approx \frac{1}{2}h((y_0 + y_n) + 2(y_1 + y_2 + \cdots + y_{n-1}))$$

$$\text{where } h = \frac{b-a}{n}$$

PHOTON NOTE

n is the number of strips, so there are $n + 1$ values of y . Concave-up curves give an overestimate.

5.11 HL only

Standard integrals

$$\int \frac{1}{x} \, dx = \ln |x| + C$$

$$\int \sin x \, dx = -\cos x + C$$

$$\int \cos x \, dx = \sin x + C$$

$$\int \frac{1}{\cos^2 x} \, dx = \tan x + C$$

$$\int e^x \, dx = e^x + C$$

5.12 HL only

Area of region enclosed by a curve and x or y -axes

$$A = \int_a^b |y| \, dx \quad \text{or} \quad A = \int_a^b |x| \, dy$$

Volume of revolution about x or y -axes

$$V = \int_a^b \pi y^2 \, dx \quad \text{or} \quad V = \int_a^b \pi x^2 \, dy$$

5.13 HL only

Acceleration

$$a = \frac{dv}{dt} = \frac{d^2s}{dt^2} = v \frac{dv}{ds}$$

PHOTON NOTE

Use $v \frac{dv}{ds}$ when acceleration is given in terms of displacement.

Distance travelled from t_1 to t_2

$$\text{distance} = \int_{t_1}^{t_2} |v(t)| \, dt$$

Displacement from t_1 to t_2

$$\text{displacement} = \int_{t_1}^{t_2} v(t) \, dt$$

5.16 HL only

Euler's method

$$y_{n+1} = y_n + h \times f(x_n, y_n); \quad x_{n+1} = x_n + h$$

where h is a constant (step length)

Euler's method for coupled systems

$$x_{n+1} = x_n + h \times f_1(x_n, y_n, t_n), \quad y_{n+1} = y_n + h \times f_2(x_n, y_n, t_n), \quad t_{n+1} = t_n + h$$

where h is a constant (step length)

5.17 HL only

Exact solution for coupled linear differential equations

$$\mathbf{x} = Ae^{\lambda_1 t} \mathbf{p}_1 + Be^{\lambda_2 t} \mathbf{p}_2$$

PHOTON NOTE

λ_1, λ_2 are the eigenvalues and $\mathbf{p}_1, \mathbf{p}_2$ the eigenvectors of the system's matrix.

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Tangent and normal at $x = a$

$$y - f(a) = f'(a)(x - a), \quad m_{\text{normal}} = -\frac{1}{f'(a)}$$

Stationary points

$$f'(x) = 0$$

Maximum or minimum: check the sign of f' either side, or use the graph on the calculator.

Separating variables (HL)

$$\frac{dy}{dx} = f(x)g(y) \Rightarrow \int \frac{1}{g(y)} dy = \int f(x) dx$$

Phase portraits (HL)

λ_1, λ_2 real, same sign: node; opposite signs: saddle; complex: spiral

Negative real parts: stable. Purely imaginary eigenvalues: closed loops (centre).

Second-order equations (HL)

$$\ddot{x} + a\dot{x} + bx = 0 \Rightarrow \dot{x} = y, \quad \dot{y} = -ay - bx$$

Rewrite as a coupled system, then use Euler's method or eigenvalues.

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