

Number and algebra

n th term of an arithmetic sequence
 $u_n = u_1 + (n - 1)d$
($n-1$)d, not nd
sum of n terms of an arithmetic sequence
 $S_n = \frac{n}{2}(2u_1 + (n - 1)d)$; $S_n = \frac{n}{2}(u_1 + u_n)$
second form if last term known

n th term of a geometric sequence
 $u_n = u_1 r^{n-1}$
sum of n terms of a finite geometric sequence
 $S_n = \frac{u_1(r^n - 1)}{r - 1} = \frac{u_1(1 - r^n)}{1 - r}$, $r \neq 1$
use the $r > 1$ form to avoid negatives

Compound interest
 $FV = PV \times \left(1 + \frac{r}{100k}\right)^{kn}$
 k = compounding periods per year
Exponents and logarithms
 $a^x = b \iff x = \log_a b$
a log undoes a power

Percentage error
 $\epsilon = \left| \frac{v_A - v_E}{v_E} \right| \times 100\%$
divide by the exact value

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Laws of exponents
 $a^m \times a^n = a^{m+n}$, $\frac{a^m}{a^n} = a^{m-n}$, $(a^m)^n = a^{mn}$
Depreciation
 $FV = PV \left(1 - \frac{r}{100}\right)^n$

Functions

Equations of a straight line
 $y = mx + c$; $ax + by + d = 0$; $y - y_1 = m(x - x_1)$
perpendicular: $m_1 m_2 = -1$
Gradient formula
 $m = \frac{y_2 - y_1}{x_2 - x_1}$
Axis of symmetry of the graph of a quadratic function
 $f(x) = ax^2 + bx + c \Rightarrow$
axis of symmetry is $x = -\frac{b}{2a}$
vertex x -coordinate

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Gradients of perpendicular lines
 $m_1 m_2 = -1$
Exponential model
 $f(x) = ka^x + c$
Direct and inverse variation
 $y = kx^n$, $y = \frac{k}{x^n}$

Geometry and trigonometry

Area of a parallelogram
 $A = bh$
Area of a triangle
 $A = \frac{1}{2}(bh)$
Area of a trapezoid
 $A = \frac{1}{2}(a + b)h$
Area of a circle
 $A = \pi r^2$
Circumference of a circle
 $C = 2\pi r$
Volume of a cuboid
 $V = lwh$
Volume of a cylinder
 $V = \pi r^2 h$
Volume of a prism
 $V = Ah$
Area of the curved surface of a cylinder
 $A = 2\pi rh$

Distance between two points (x_1, y_1) and (x_2, y_2)
 $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
Midpoint of a line segment
 $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
Distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2)
 $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$
Midpoint of a line segment
 $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$
Volume of a right-pyramid
 $V = \frac{1}{3}Ah$
Volume of a right cone
 $V = \frac{1}{3}\pi r^2 h$
Area of the curved surface of a cone
 $A = \pi rl$
 l = slant height

Volume of a sphere
 $V = \frac{4}{3}\pi r^3$
Surface area of a sphere
 $A = 4\pi r^2$
Sine rule
 $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
check the ambiguous case
Cosine rule
 $c^2 = a^2 + b^2 - 2ab \cos C$; $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$
three sides, or SAS
Area of a triangle
 $A = \frac{1}{2}ab \sin C$
 C is between a and b
Length of an arc (degrees)
 $l = \frac{\theta}{360} \times 2\pi r$
degrees (radians in 3.7)
Area of a sector (degrees)
 $A = \frac{\theta}{360} \times \pi r^2$

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Gradient of a perpendicular bisector
 $m_{\perp} = -\frac{1}{m}$
Bearings (clockwise from north)
 045° , 120° , 300°

Statistics and probability

Interquartile range
 $IQR = Q_3 - Q_1$
outlier: $1.5 \times IQR$ beyond Q_1 , Q_3
Mean, $\bar{x}$, of a set of data
 $\bar{x} = \frac{\sum_{i=1}^n f_i x_i}{n}$
Probability of an event A
 $P(A) = \frac{n(A)}{n(U)}$
Complementary events
 $P(A) + P(A') = 1$
at least one = $1 - P(\text{none})$
Combined events
 $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
Mutually exclusive events
 $P(A \cup B) = P(A) + P(B)$
Conditional probability
 $P(A|B) = \frac{P(A \cap B)}{P(B)}$
given B: divide by $P(B)$

Independent events
 $P(A \cap B) = P(A)P(B)$
use it to test independence
Expected value of a discrete random variable X
 $E(X) = \sum_{i=1}^k x_i P(X = x_i)$
fair game: $E = 0$
Binomial distribution $X \sim B(n, p)$
Mean
 $E(X) = np$
Variance
 $\text{Var}(X) = np(1 - p)$
 $SD = \sqrt{\text{variance}}$

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Standardized normal variable
 $z = \frac{x - \mu}{\sigma}$
Expected frequency (χ^2 test)
expected = $\frac{\text{row total} \times \text{column total}}{\text{grand total}}$
Degrees of freedom (χ^2 test for independence)
 $\nu = (r - 1)(c - 1)$
Outliers
 $x < Q_1 - 1.5 \times IQR$ or $x > Q_3 + 1.5 \times IQR$

Calculus

Derivative of x^n
 $f(x) = x^n \Rightarrow f'(x) = nx^{n-1}$
roots and fractions $\rightarrow$ powers
Integral of x^n
 $\int x^n dx = \frac{x^{n+1}}{n+1} + C$, $n \neq -1$
never drop $+C$
trapezoidal rule
 $\int_a^b y dx \approx \frac{1}{2}h((y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1}))$
 n strips, $n+1$ values

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Tangent and normal at $x = a$
 $y - f(a) = f'(a)(x - a)$, $m_{\text{normal}} = -\frac{1}{f'(a)}$
Stationary points
 $f'(x) = 0$