

Mathematical equations

Area of a triangle  
 $A = \frac{1}{2}(bh)$   
Area of a circle  
 $A = \pi r^2$   
Circumference of a circle  
 $C = 2\pi r$   
Volume of a cuboid  
 $V = lwh$   
Volume of a cylinder  
 $V = \pi r^2 h$   
Volume of a prism  
 $V = Ah$   
Volume of a sphere  
 $V = \frac{4}{3}\pi r^3$   
Area of the curved surface of a cylinder  
 $A = 2\pi rh$

Vectors  
 $A_H = A \cos \theta$      $A_V = A \sin \theta$   
Trigonometric relationships  
 $\tan \theta = \frac{\sin \theta}{\cos \theta}$      $\sin^2 \theta + \cos^2 \theta = 1$   
**NOT IN THE IB BOOKLET**  
Surface area of a sphere  
 $A = 4\pi r^2$   
Small-angle approximation  
 $\sin \theta \approx \tan \theta \approx \theta$  (radians)

Uncertainties

$y = a \pm b \Rightarrow \Delta y = \Delta a + \Delta b$   
 $y = \frac{ab}{c} \Rightarrow \frac{\Delta y}{y} = \frac{\Delta a}{a} + \frac{\Delta b}{b} + \frac{\Delta c}{c}$   
 $y = a^n \Rightarrow \frac{\Delta y}{y} = \left| n \frac{\Delta a}{a} \right|$

Fundamental constants

Acceleration  $g$  9.8 m s<sup>-2</sup> (Earth's surface) of free fall  
Gravitational  $G$  6.67 × 10<sup>-11</sup> N m<sup>2</sup> kg<sup>-2</sup> constant  
Avogadro  $N_A$  6.02 × 10<sup>23</sup> mol<sup>-1</sup> constant  
Gas  $R$  8.31 J K<sup>-1</sup> mol<sup>-1</sup> constant  
Boltzmann  $k_B$  1.38 × 10<sup>-23</sup> J K<sup>-1</sup> constant  
Stefan–Boltzmann  $\sigma$  5.67 × 10<sup>-8</sup> W m<sup>-2</sup> K<sup>-4</sup> constant  
Coulomb  $k$  8.99 × 10<sup>9</sup> N m<sup>2</sup> C<sup>-2</sup> constant  
Permittivity  $\epsilon_0$  8.85 × 10<sup>-12</sup> C<sup>2</sup> N<sup>-1</sup> m<sup>-2</sup> of free space  
Permeability  $\mu_0$  4π × 10<sup>-7</sup> T m A<sup>-1</sup> of free space  
Speed of light in vacuum  $c$  3.00 × 10<sup>8</sup> m s<sup>-1</sup>  
Planck  $h$  6.63 × 10<sup>-34</sup> J s constant  
Elementary charge  $e$  1.60 × 10<sup>-19</sup> C  
Electron rest mass  $m_e$  9.11 × 10<sup>-31</sup> kg = 0.000549 u = 0.511 MeV c<sup>-2</sup>  
Proton rest mass  $m_p$  1.673 × 10<sup>-27</sup> kg = 1.007276 u = 938 MeV c<sup>-2</sup>  
Neutron rest mass  $m_n$  1.675 × 10<sup>-27</sup> kg = 1.008665 u = 940 MeV c<sup>-2</sup>  
(Unified) atomic mass unit  $u$  1.661 × 10<sup>-27</sup> kg = 931.5 MeV c<sup>-2</sup>

Solar constant  $S$  1.36 × 10<sup>3</sup> W m<sup>-2</sup>  
Fermi radius  $R_0$  1.20 × 10<sup>-15</sup> m

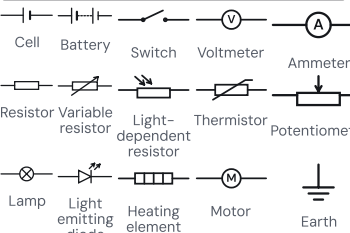
Metric (SI) multipliers

peta P 10<sup>15</sup> · tera T 10<sup>12</sup> · giga G 10<sup>9</sup> · mega M 10<sup>6</sup> · kilo k 10<sup>3</sup> · hecto h 10<sup>2</sup> · deca da 10<sup>1</sup> · deci d 10<sup>-1</sup> · centi c 10<sup>-2</sup> · milli m 10<sup>-3</sup> · micro μ 10<sup>-6</sup> · nano n 10<sup>-9</sup> · pico p 10<sup>-12</sup> · femto f 10<sup>-15</sup>

Unit conversions

1 radian (rad) =  $\frac{180^\circ}{\pi}$   
Temperature (K) = temperature (°C) + 273  
1 light year (ly) = 9.46 × 10<sup>15</sup> m  
1 parsec (pc) = 3.26 ly  
1 astronomical unit (AU) = 1.50 × 10<sup>11</sup> m  
1 kilowatt-hour (kWh) = 3.60 × 10<sup>6</sup> J  
 $hc = 1.99 \times 10^{-25} \text{ J m} = 1.24 \times 10^{-6} \text{ eV m}$

Circuit symbols



Electromagnetic spectrum

γ rays · X-rays · UV · visible (400–700 nm) · IR · microwaves · radio, from short wavelength (high energy) to long. Scale 10<sup>-16</sup> m to 10<sup>8</sup> m.

A. Space, time and motion

A.1 Kinematics

$s = \frac{u+v}{2}t$   
constant a only (all four)  
 $v = u + at$   
 $s = ut + \frac{1}{2}at^2$   
 $v^2 = u^2 + 2as$   
no t in it

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Projectiles

$v_x = u \cos \theta$  (constant)     $v_y = u \sin \theta - gt$

A.2 Forces and momentum

$F_f \leq \mu_s F_N$   
static: up to  $\mu_s F_N$   
 $F_f = \mu_d F_N$   
 $F_H = -kx$   
 $F_d = 6\pi\eta rv$   
small sphere, laminar flow  
 $F_b = \rho Vg$   
ρ of the fluid  
 $F_g = mg$   
 $p = mv$   
 $J = F\Delta t$   
area under F–t  
 $F = ma = \frac{\Delta p}{\Delta t}$   
 $a = \frac{v^2}{r} = \omega^2 r = \frac{4\pi^2 r}{T^2}$   
points to the centre  
 $v = \frac{2\pi r}{T} = \omega r$

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Conservation of momentum  
 $m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$   
Terminal velocity (sphere in fluid)  
 $mg = F_b + 6\pi\eta r v_t$

A.3 Work, energy and power

$W = Fs \cos \theta$   
θ: force vs displacement  
 $E_k = \frac{1}{2}mv^2 = \frac{p^2}{2m}$   
 $\Delta E_p = mg\Delta h$   
 $E_H = \frac{1}{2}k\Delta x^2$   
area under F–x  
 $P = \frac{\Delta W}{\Delta t} = Fv$   
 $\eta = \frac{\text{useful work out}}{\text{total work in}} = \frac{\text{useful power out}}{\text{total power in}}$

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Work–energy principle  
 $W_{\text{net}} = \Delta E_k$   
Energy density  
energy density =  $\frac{\text{energy}}{\text{volume}}$

A.4 Rigid body mechanics HL

$\tau = Fr \sin \theta$   
θ between F and r  
 $\Delta \theta = \frac{\omega_f + \omega_i}{2}t$   
 $\omega_f = \omega_i + \alpha t$   
 $\Delta \theta = \omega_i t + \frac{1}{2}\alpha t^2$   
 $\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$   
 $I = \Sigma mr^2$   
 $\tau = I\alpha$   
 $L = I\omega$   
 $\Delta L = \tau\Delta t$   
 $\Delta L = \Delta(I\omega)$   
no torque: Iω constant  
 $E_k = \frac{1}{2}I\omega^2 = \frac{L^2}{2I}$

A.5 Galilean and special relativity HL

$x' = x - vt$   
 $t' = t$   
 $u' = u - v$   
 $x' = \gamma(x - vt)$  where  $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$   
 $\gamma \geq 1$   
 $t' = \gamma \left( t - \frac{vx}{c^2} \right)$   
 $u' = \frac{u - v}{1 - \frac{uv}{c^2}}$   
 $(\Delta s)^2 = (c\Delta t)^2 - \Delta x^2$   
same in every frame  
 $\Delta t = \gamma\Delta t_0$   
t<sub>0</sub> = proper time  
 $L = \frac{L_0}{\gamma}$   
L<sub>0</sub> in the rest frame  
 $\tan \theta = \frac{v}{c}$

B. The particulate nature of matter

B.1 Thermal energy transfers

$\rho = \frac{m}{V}$

$\bar{E}_k = \frac{3}{2}k_B T$   
T in kelvin  
 $Q = mc\Delta T$   
 $Q = mL$   
no temperature change  
 $\frac{\Delta Q}{\Delta t} = kA \frac{\Delta T}{\Delta x}$   
k = conductivity  
 $L = \sigma AT^4$   
L = power; A = 4πR<sup>2</sup>  
 $b = \frac{L}{4\pi d^2}$   
inverse square  
 $\lambda_{\text{max}} T = 2.9 \times 10^{-3} \text{ m K}$   
hotter → shorter λ

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Black-body radius of a star

$R = \sqrt{\frac{L}{4\pi\sigma T^4}}$

B.2 Greenhouse effect

emissivity =  $\frac{\text{power radiated per unit area}}{\sigma T^4}$   
albedo =  $\frac{\text{total scattered power}}{\text{total incident power}}$

Earth ≈ 0.3

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Power radiated by a real surface  
 $P = e\sigma AT^4$   
Mean incoming intensity on Earth  
 $\frac{S}{4}$

B.3 Gas laws

$P = \frac{F}{A}$   
 $n = \frac{N}{N_A}$   
 $\frac{PV}{T} = \text{constant}$   
kelvin  
 $PV = nRT = Nk_B T$   
 $P = \frac{1}{3}\rho v^2$   
mean square speed  
 $U = \frac{3}{2}nRT = \frac{3}{2}Nk_B T$

B.5 Current and circuits

$I = \frac{\Delta q}{\Delta t}$   
 $V = \frac{W}{q}$   
 $R = \frac{V}{I}$   
 $\rho = \frac{RA}{L}$   
property of the material  
 $P = IV = I^2 R = \frac{V^2}{R}$   
Series  
 $I = I_1 = I_2 = \dots$      $V = V_1 + V_2 + \dots$   
 $R_s = R_1 + R_2 + \dots$   
Parallel  
 $I = I_1 + I_2 + \dots$      $V = V_1 = V_2 = \dots$   
 $\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \dots$   
parallel always lowers R  
 $\epsilon = I(R + r)$   
r = internal resistance

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Terminal potential difference  
 $V = \varepsilon - Ir$

Potential divider  
 $V_{\text{out}} = V_{\text{in}} \frac{R_2}{R_1 + R_2}$

## B.4 Thermodynamics HL

$Q = \Delta U + W$

$W$  done BY the gas

$W = P\Delta V$

constant P only

$\Delta U = \frac{3}{2}nR\Delta T = \frac{3}{2}Nk_B\Delta T$

$\Delta S = \frac{\Delta Q}{T}$

$S = k_B \ln \Omega$

$PV^{\frac{5}{3}} = \text{constant}$

adiabatic, monatomic

$\eta = \frac{\text{useful work}}{\text{input energy}}$

$\eta_{\text{Carnot}} = 1 - \frac{T_c}{T_h}$

the maximum; kelvin

## C. Wave behaviour

### C.1 Simple harmonic motion

$a = -\omega^2 x$

defines SHM

$T = \frac{1}{f} = \frac{2\pi}{\omega}$

$T = 2\pi\sqrt{\frac{m}{k}}$

$T = 2\pi\sqrt{\frac{l}{g}}$

small angles; mass irrelevant

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Angular frequency  
 $\omega = 2\pi f$

### C.2 Wave model

$v = f\lambda = \frac{\lambda}{T}$

$f$  set by the source

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Intensity and amplitude  
 $I \propto A^2$   
Inverse-square law  
 $I \propto \frac{1}{r^2}$

### C.3 Wave phenomena

$\frac{n_1}{n_2} = \frac{\sin \theta_2}{\sin \theta_1} = \frac{v_2}{v_1}$

angles from the normal

Constructive interference

path difference =  $n\lambda$

Destructive interference

path difference =  $(n + \frac{1}{2})\lambda$

$s = \frac{\lambda D}{d}$

$d$  = slit separation

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Critical angle  
 $\sin c = \frac{n_2}{n_1}$

### C.4 Standing waves and resonance

No equations in the booklet.

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String fixed at both ends, or pipe open at both ends  
 $\lambda_n = \frac{2L}{n}, \quad f_n = \frac{nv}{2L} \quad (n = 1, 2, 3, \dots)$   
Pipe closed at one end  
 $\lambda_n = \frac{4L}{n}, \quad f_n = \frac{nv}{4L} \quad (n = 1, 3, 5, \dots)$

### C.5 Doppler effect

$\frac{\Delta f}{f} = \frac{\Delta \lambda}{\lambda} \approx \frac{v}{c}$

redshift: moving away

### C.1 Simple harmonic motion HL

$x = x_0 \sin(\omega t + \phi)$

$v = \omega x_0 \cos(\omega t + \phi)$

$v = \pm \omega \sqrt{x_0^2 - x^2}$

max speed  $\omega x_0$  at  $x = 0$

$E_T = \frac{1}{2}m\omega^2 x_0^2$

$E_p = \frac{1}{2}m\omega^2 x^2$

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Kinetic energy in SHM  
 $E_k = \frac{1}{2}m\omega^2 (x_0^2 - x^2)$

### C.3 Wave phenomena HL

$\theta = \frac{\lambda}{b}$

radians; first minimum

$n\lambda = d \sin \theta$

$d = 1 / \text{lines per metre}$

### C.5 Doppler effect HL

Moving source

$f' = f \left( \frac{v}{v \pm u_s} \right)$

approaching: minus

Moving observer

$f' = f \left( \frac{v \pm u_o}{v} \right)$

approaching: plus

## D. Fields

### D.1 Gravitational fields

$F = G \frac{m_1 m_2}{r^2}$

$r$  centre to centre

$g = \frac{F}{m} = G \frac{M}{r^2}$

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Kepler's third law (circular orbit)  
 $T^2 = \frac{4\pi^2}{GM} r^3$

### D.2 Electric and magnetic fields

$F = k \frac{q_1 q_2}{r^2}$  where  $k = \frac{1}{4\pi\epsilon_0}$

$E = \frac{F}{q}$

$E = \frac{V}{d}$

uniform field only

### D.3 Motion in electromagnetic fields

$F = qvB \sin \theta$

flip for negative charge

$F = BIL \sin \theta$

$\frac{F}{L} = \mu_0 \frac{I_1 I_2}{2\pi r}$

same direction: attract

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Radius of a charge in a magnetic field  
 $r = \frac{mv}{qB}$   
Velocity selector  
 $v = \frac{E}{B}$

### D.1 Gravitational fields HL

$E_p = -G \frac{m_1 m_2}{r}$

zero at infinity

$V_g = -G \frac{M}{r}$

$g = -\frac{\Delta V_g}{\Delta r}$

minus the gradient

$W = m\Delta V_g$

$v_{\text{esc}} = \sqrt{\frac{2GM}{r}}$

$v_{\text{orbital}} = \sqrt{\frac{GM}{r}}$

independent of satellite mass

**NOT IN THE IB BOOKLET**  
Total energy of an orbit  
 $E_{\text{total}} = -\frac{GMm}{2r}$

### D.2 Electric and magnetic fields HL

$E_p = k \frac{q_1 q_2}{r}$

$V_e = \frac{kQ}{r}$

scalar: add with signs

$E = -\frac{\Delta V_e}{\Delta r}$

$W = q\Delta V_e$

### D.4 Induction HL

$\Phi = BA \cos \theta$

$\theta$  to the normal

$\varepsilon = -N \frac{\Delta \Phi}{\Delta t}$

minus sign = Lenz's law

$\varepsilon = BvL$

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Flux linkage  
 $N\Phi$

## E. Nuclear and quantum physics

### E.1 Structure of the atom

$E = hf$

$\approx$  gap between levels

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Photon energy from wavelength  
 $E = \frac{hc}{\lambda}$

### E.3 Radioactive decay

$E = mc^2$

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Binding energy from the mass defect  
 $\Delta E = \Delta m c^2$   
Whole half-lives  
 $N = N_0 \left(\frac{1}{2}\right)^n$

### E.5 Fusion and stars

$d \text{ (parsec)} = \frac{1}{p \text{ (arc-second)}}$

nearby stars only

### E.1 Structure of the atom HL

$R = R_0 A^{\frac{1}{3}}$

same density for all nuclei

$E = -\frac{13.6}{n^2} \text{ eV}$

hydrogen only

$mvr = \frac{nh}{2\pi}$

### E.2 Quantum physics HL

$E_{\text{max}} = hf - \Phi$

brighter  $\neq$  more energetic

$\lambda = \frac{h}{p}$

$\lambda_f - \lambda_i = \Delta \lambda = \frac{h}{m_e c} (1 - \cos \theta)$

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Threshold frequency  
 $f_0 = \frac{\Phi}{h}$   
Stopping potential  
 $eV_s = E_{\text{max}}$

### E.3 Radioactive decay HL

$N = N_0 e^{-\lambda t}$

$\lambda$  = decay constant

$A = \lambda N = \lambda N_0 e^{-\lambda t}$

$T_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$